R&D NOTE

Percolation Modeling of Foam Generation in Porous Media

William R. Rossen, Jianxin Shi, and Sabine C. Zeilinger
Dept. of Petroleum Engineering, The University of Texas at Austin, Austin, TX 78712

A foam is a dispersion of a large volume of gas in a continuous liquid phase, stabilized by surfactant. Foams can improve sweep efficiency and oil recovery in gas-injection enhanced oil recovery projects (Hirasaki, 1989a,b; Smith, 1988; Rossen, 1994). An important issue for these foam processes is the ease of foam generation in porous media. Previously, Rossen and Gauglitz (1990) derived a percolation model for foam generation in steady gas-liquid flow in porous media. More recent advances in percolation theory require modification of this model, as described in this note.

Briefly, Rossen and Gauglitz assumed that the liquid films or lamellae present in foam block a randomly selected fraction $(1-f)$ of the pore throats in the medium. Creating the large number of lamellae that define a foam from the relatively few present initially requires displacing these lamellae from their pore throats so they can multiply by the processes of lamella division and repeated snapoff (Ransohoff and Radke, 1988; Rossen, 1994). Displacing lamellae from pore throats to initiate the generation process requires imposing a pressure difference across the throat of order one or a few kPa (a few tenths of a psi). "Generating" foam, therefore, depends on lamella mobilization, which depends on the magnitude of the pressure drop ΔP across individual pore throats blocked by lamellae. Rossen and Gauglitz's model predicts that the minimum pressure gradient for foam generation ∇P^{min} decreases nearly linearly as f approaches f_c, the percolation threshold for the pore network, from either direction.

Discontinuous Gas Foam

There are two cases, depending on the value of f. If f, the fraction of throats *not* initially blocked by lamellae, is less than the percolation threshold f_c for the pore network (Stauffer, 1992), then no gas can flow unless some of these lamellae are displaced. (We use f and f_c here, rather than the customary percolation symbols p and p_c (Stauffer, 1992), to avoid confusion with pressure and capillary pressure.) Since for $f<f_c$, the gas phase is broken into discrete bubbles, displacing *any* lamella requires displacing a *train* of bubbles that spans the pore network. This train represents the unique path through the network with the minimum ∇P required to overcome the capillary resistance of its lamellae to displacement. Rossen and Gauglitz (1990) presented an approximate solution for ∇P^{min} in a Bethe (or Cayley) tree network. The results are shown on the lefthand side of Figure 1 for the parameter values given by Rossen and Gauglitz. For this case, $f_c=0.25$.

More recently, Rossen and Mamun (1993) presented a recursive formula for ∇P^{min} on a Bethe tree lattice for $f<f_c$ (see also Stinchcombe et al., 1986). For the same parameter values, the result is shown on the lefthand side of Figure 2. ∇P^{min} is roughly as predicted by Rossen and Gauglitz for $f<f_c$.

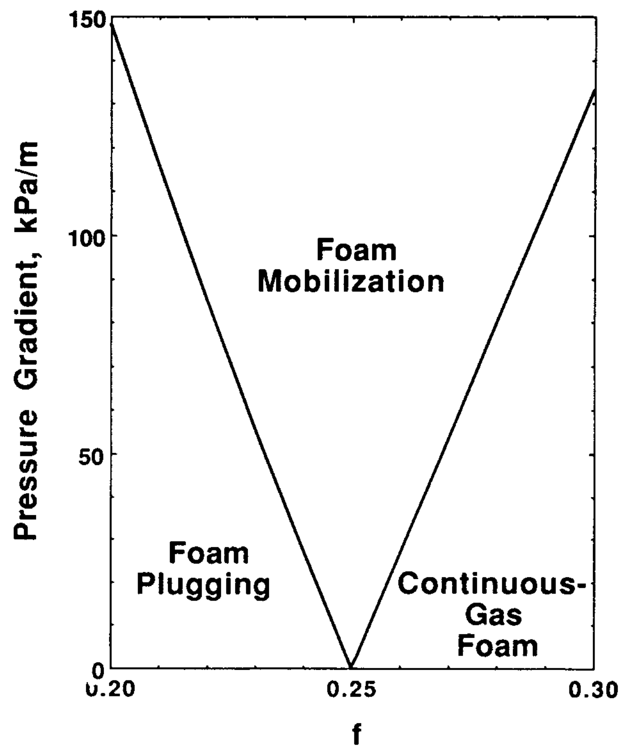

Figure 1. Model of Rossen and Gauglitz (1990) for foam generation on Bethe tree network.

Correspondence concerning this work should be addressed to W. R. Rossen.

Continuous Gas Foam

If $f>f_c$, then there is continuous gas flow around the pore throats blocked by lamellae. Creating a foam with the enormous gas-flow resistances observed in the laboratory requires displacing these lamellae so they can multiply by lamellae division and snapoff. The lamellae most likely to be mobilized in this case are those that block large, deadend clusters of pores singly connected to the flowing "backbone" of interconnected gas-filled pores (Rossen and Gauglitz, 1990). If one assumes that ∇P is roughly uniform in the pore network, then the ΔP across a lamella blocking a deadend cluster is proportional to the length of the cluster in the direction of flow. Rossen and Gauglitz presented an approximate solution for this cluster size, based on the size of the *average* blocked cluster. The resulting prediction of ∇P^{min} is shown on the righthand side of Figure 1.

However, although the size of the *average* blocked cluster diverges in magnitude as $f \rightarrow f_c$, the *number* of these clusters, and therefore the number of candidate sites for lamella mobilization, approaches zero as $f \rightarrow f_c$ (Rossen, 1991). Mobilizing a single lamella is not sufficient for successful foam generation, since a lamella may rupture upon displacement. Instead, some small *population* of lamellae must be mobilized to overcome these losses and trigger the massive lamella-creation processes observed experimentally (Rossen and Gauglitz, 1990). Therefore, the characteristic cluster size is not that of the average blocked cluster, averaged over a shrinking population as f approaches f_c, but the cluster size that occurs with some given frequency in the pore network: say, a ratio of one throat in 1,000 or 10,000. For instance, Rossen and Gauglitz's original model corresponds to a fraction of roughly one throat in 3,000 for f far above f_c (Rossen, 1991). The exact fraction of lamella-mobilization sites on the network required for successful foam generation depends on the likelihood a lamella will rupture upon mobilization (Gauglitz, 1993), which is outside the scope of the percolation theory. Rossen (1991) solved for the characteristic cluster size, as a function of required frequency of occurrence in the network, for the Bethe tree network: it increases at first as f decreases toward f_c, but it abruptly drops to zero for some $f>f_c$, due to the shrinking overall population of blocked clusters. As a result, as shown in Figure 2 for the same parameter values as in Figure 1, ∇P^{min} abruptly diverges in magnitude at some $f>f_c$.

It is difficult to measure f directly in porous media. More commonly, one measures the minimum gas injection velocity for foam generation as a function of injected liquid volume fraction (or foam quality) in steady flow. Using the parameter values in Rossen and Gauglitz (1990), however, the difference between the original and revised models for $f>f_c$ so evident in comparing Figures 1 and 2 occurs at values of liquid volume fraction greater than 0.95, which is higher than values of experimental interest.

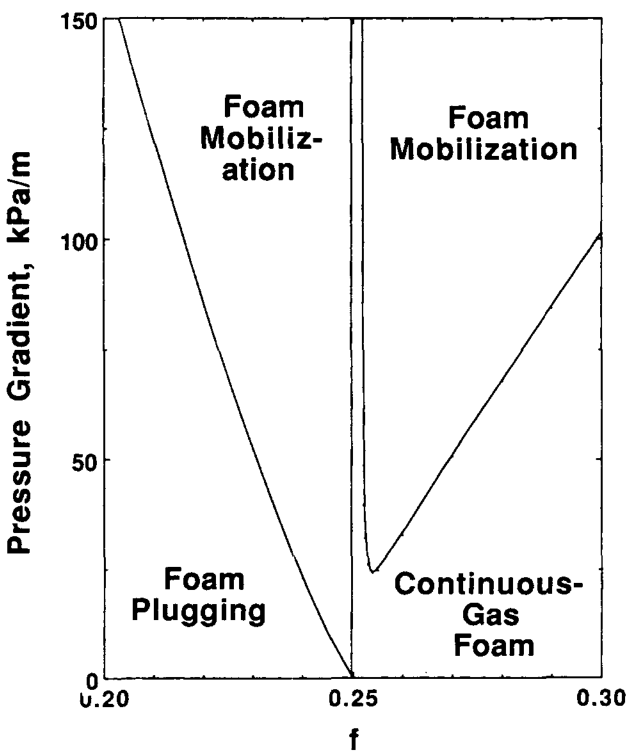

Figure 2. Revised model for foam mobilization on Bethe tree network.

Discussion

The lack of convergence of the two models in Figure 2 as f approaches f_c is disquieting. We believe the error lies in the model for $f>f_c$. Rossen and Gauglitz's (1990) original model for $f>f_c$ made two problematic assumptions: first, ∇P is roughly uniform across the pore network; second, it is the size of the average blocked cluster that controls ∇P^{min}. Our revised model corrects only the second assumption.

We believe the divergence of ∇P^{min} predicted by this model as $f \rightarrow f_c$ from above derives from the breakdown of the first assumption as $f \rightarrow f_c$. For f very near f_c, large ΔP, initiating foam mobilization, can occur across small blocked clusters, especially in the vicinity of so-called "red" bonds, bottlenecks in the tortuous flow path (Pike and Stanley, 1981; Coniglio, 1982; Stanley and Coniglio, 1983; Kahng et al., 1987). Thus, foam mobilization does not depend on the existence of large blocked gas clusters as $f \rightarrow f_c$.

Verifying this conjecture will require large Monte Carlo studies on two- and three-dimensional networks. It is in part to motivate such studies that we present these results.

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Notation

- f = fraction of pore throats open for gas flow, not blocked by lamellae
- f_c = percolation threshold
- ΔP = pressure difference across a pore throat
- ∇P = macroscopic pressure gradient in medium
- ∇P^{min} = minimum ∇P to mobilize lamellae in medium

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